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Numerical modeling of phenomena of waterhammer using a model of fluid–structure interaction

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ABSTRACT

We present a numerical code for fluid–structure interactions to solve the problem of waterhammer in pipes with thin walls. The pipe is modeled by planar beams theory of Bernoulli–Euler in longitudinal and transverse vibrations. This code is the coupling of the finite element method combined with the Newmark algorithm for movement of the pipe wall, and, for the fluid, the method of characteristics. Unlike the classical theory, this code illustrates the side effects of fluid–structure interaction affecting parameters of waterhammer in elastic and viscoelastic pipe.

RÉSUMÉ

On présente un code numérique d'interactions fluide–structure pour résoudre le problème de coup de bélier en conduites à paroi mince. La conduite est modélisée par la théorie des poutres planes de Bernoulli–Euler en vibrations longitudinale et transversale. Ce code est le couplage de la méthode des éléments finis associée à l'algorithme de Newmark pour le mouvement de la paroi de la conduite et, pour le fluide, à la méthode des caractéristiques. Contrairement à la théorie classique, ce code permet d'illustrer les effets secondaires d'interaction fluide–structure affectant les paramètres de coup de bélier dans les cas de conduite élastique et viscoélastique.

1. Introduction

The classic study of the phenomena of waterhammer in pipes is usually conducted in one-dimensional flow and the conduit is supposed to be deformed instantly as if it consists of a stacking of rings without mass [1–3]. A less restrictive assumption and taking into account the dynamic coupling effect of the pipe wall through the Poisson's ratio of material has been developed by Wiggert et al. [4], Otwell [5], Chaudhry et al. [6,7]. Bahrar et al. [8,9] have developed a numeric code using the shell theory of Timoshenko for dynamic behavior of pipe wall and bi-dimensional flow. More recently, Wiggert and Tijsseling [10] have conducted similar studies, but in the assumptions of Timoshenko beams.

The work presented herein has been done, in planar beams theory of Bernoulli–Euler [11,12], for pipe in transverse and longitudinal vibrations. It discusses the effects of dynamic coupling of the pipe wall and the fluid and illustrates the parameters affecting waterhammer and evaluates the approximations made in the classical theory.

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Nomenclature

a	Celerity of a classical waterhammer
α	Parameter characterizing the type of anchoring the pipe
A_f	Cross section area of the fluid
A_p	Cross sectional area of pipe wall
$Cars(J)$	The Laplace–Carson transform
$\mathrm{d}()/dt$	Total derivative
$\delta()/\delta t$	Derivative operator along characteristic curves
$\partial()/\partial t$	Temporal derivative operator
$(\dot{})$	Time derivative
$(')$	Spacial derivative
D	Inner diameter of the pipe
D_m	Average diameter
e	Pipe wall thickness
ε^e	The instantaneous elastic deformation
ε^r	Strain retarded creep
E	Young's modulus
$E(0)$	The instantaneous relaxation modulus
E_i	The generic spring of Kelvin–Voigt model
E^*	Differential operator associated to the relaxation modulus of the material
I	Moment of inertia of the cross section of the pipe (fluid and material pipe)
g	Acceleration of gravity
$J(0)$	The instantaneous compliance of the material of pipe wall
J	The creep compliance function
J_i	The generic compliance of the Kelvin–Voigt element
κ	Bulk modulus of elasticity of fluid
L	Length of pipe wall
m_f	Mass of fluid per unit of length
m_p	Mass of pipe wall material per unit of length
m	Total mass of fluid and pipe wall per unit length
$N_e(x)$	The shape functions
P	Fluid average pressure along a cross section
q_e	Elementary vector of nodal degree of freedom
s	Complex variable
t	Time
τ, τ_i	Relaxation times
η, η_i	Viscous dampers
T_f	Friction term of fluid at inner surface of pipe wall
u, w	Longitudinal and transverse displacements of pipe wall
V	Fluid average velocity
x	Longitudinal component of axis along a pipe
z	The piezometric head at the abscissa x
ρ_f	Mass density of fluid
ρ_m	Mass density of pipe wall material
ν	Poisson's ratio of material
$\dot{u}$	Longitudinal velocity of the pipe wall
$[m]$	Elementary mass matrix
$[c]$	Elementary damping matrix
$[k]$	Elementary stiffness matrix
$\{f\}$	Elementary term reflecting the transfer of the momentum
$[M]$	Global mass matrix
$[C]$	Global damping matrix
$[K]$	Global stiffness matrix
$\{F\}$	Global term reflecting the transfer of the momentum
$X(t)$	Vector function of displacements of the pipe wall at nodes

2. Basic equations and assumptions made

The basic equations are derived from the classical laws of conservation of mass, momentum for the fluid and the pipe wall in the isentropic transformations. Assume also the fluid is barotropic Newtonian and the material of the pipe wall behaves as elastic or viscoelastic model of Kelvin–Voigt [13]. Geometrically, the conduit is supposed to be horizontal cylindrical and circular. One end is rigidly attached to the tank which requires a constant pressure and the other is on the fixed support. The flow is axisymmetric and the longitudinal gradients of the flow velocity are assumed to be small compared to transverse gradients.

2.1. The equations of fluid

Given these assumptions, the averaged equations of the flow in a cross section of pipe can be written as hyperbolic system that is suitable for characteristic methods:

$$\frac{\partial(\rho_f A_f)}{\partial t} + \frac{\partial(\rho_f A_f V)}{\partial x} = 0 \tag{1}$$

$$\rho_f \frac{\mathrm{d}V}{\mathrm{d}t} + \frac{\partial P}{\partial x} + \rho_f g \frac{\partial z}{\partial x} - \frac{4T_f}{D} = 0 \tag{2}$$

If we introduce $\varepsilon^e = \alpha(p(x,t) - p(x,0))D_m J(0)/2e$ the instantaneous elastic deformation of the wall and $\varepsilon^r = \int_0^t \alpha(P(x, t-\tau) - P(x,0))\frac{D_m}{2e}\frac{\mathrm{d}J(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau$ its resulting strain retarded creep, the deformation of the wall can be considered as the sum

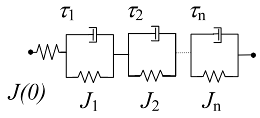

Fig. 1. The model.

of two terms $\varepsilon^e + \varepsilon^r$, and α: parameter characterizing the type of anchoring the pipe and which, in the case of a pipe anchored longitudinally, is written $\alpha = 1 + e^2/D_m^2 + 2\nu e D_m - \nu^2(1 - e/D_m)^2$ [2] and Eq. (1) above becomes:

$$\frac{1}{\rho_f}\frac{\mathrm{d}P}{\mathrm{d}t} + a^2\frac{\partial V}{\partial x} - 2a^2\nu\frac{\partial \dot{u}}{\partial x} + 2a^2\frac{\mathrm{d}\varepsilon^r}{\mathrm{d}t} = 0 \tag{1'}$$

where:

$$a = \left(\rho_f\left(1/\kappa + \alpha D_m J(0)/e\right)\right)^{-1/2}$$

The term $-2a^2\nu\frac{\partial \dot{u}}{\partial x}$ represents the coupling of fluid movement and dynamic behavior in the longitudinal direction through the Poisson's ratio ν of the pipe material.

The creep compliance function can be expressed as:

$$J(t) = J(0) + \sum_{i=1}^{n} J_i\left(1 - \exp(-t/\tau_i)\right)$$

corresponding to a generalized model of Kelvin–Voigt [13] in Fig. 1. In this representation, $J_i = 1/E_i$, E_i is the spring in parallel with viscous dampers η_i, leading to a relaxation time $\tau_i = \eta_i/E_i$, $J(0) = 1/E(0)$, $E(0)$ represents the instantaneous relaxation modulus.

2.2. *Dynamic equations of pipe wall*

Under the assumptions previously mentioned and small deformations, the dynamic equations of the pipe wall in longitudinal and transverse direction are reduced to the following system [11,12]:

$$A_p E^*\frac{\partial^2 u}{\partial x^2} - m_p\frac{\partial^2 u}{\partial t^2} + \frac{A_p D\nu}{2e}\frac{\partial P}{\partial x} + T_f A_f = 0 \tag{3}$$

$$E^* I\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + 2Vm_f\frac{\partial^2 w}{\partial x\partial t} + V^2 m_f\frac{\partial^2 w}{\partial^2 x} = 0 \tag{4}$$

where $E^* = E(1 + \tau\partial/\partial t)$ is the differential operator associated to the relaxation modulus of the material of pipe wall. We have in the Laplace–Carson transform ($Cars(f) = s\int_0^\infty f(t)e^{-st}\,\mathrm{d}t$), the relation $Cars(J) = 1/Cars(E^*)$ [13]. In elastic case, E^* is identified with Young's modulus E.

The term $\nu\frac{A_p D}{2e}\frac{\partial P}{\partial x}$, in Eq. (3), is the coupling term representing the effects of the expansion of the pipe induced by fluid pressure on the longitudinal behavior through the Poisson's ratio. In Eq. (4) the two terms, $E^* I\partial^4 w/\partial x^4$ and $m\partial^2 w/\partial t^2$, represent the bending stiffness and the inertial force, and the coupling terms, $2Vm_f\frac{\partial^2 w}{\partial x\partial t}$ and $V^2 m_f\frac{\partial^2 w}{\partial^2 x}$, describe the forces required to change the flow direction and to twist the element of fluid by the effect of the wall.

3. Initial and boundary conditions

The initial conditions are those for steady flow and the balance for the pipe wall. The boundary conditions are in addition to the pressure imposed by the tank on the upstream and the instantaneous closing of valve on the downstream, the conditions of fluid–conduit interfaces requiring, in viscous flow, equal velocities and stresses as well as:

$$u(x=0,t) = \frac{\partial u(L,t)}{\partial x} = 0$$

and

$$w(x=0,t) = \frac{\partial w(x=0,t)}{\partial x} = w(x=L,t) = \frac{\partial^2 w(x=L)}{\partial^2 x} = \frac{\partial^3 w(x=L)}{\partial x^3} = 0$$

4. Numerical solution

The numerical solution of Eqs. (1′), (2), (3) and (4) associated to initial and boundary conditions are obtained by separate treatment method of the fluid–structure interaction: at each step of time we first solve the equations for the fluid by the method of characteristics (MOF) with a regular grid [14] and then the dynamic equations of pipe wall is solved by the finite elements method with at each node three degrees of freedom. This algorithm is combined with that of temporal integration of Newmark [15]. This yield:

(a) For the fluid along the characteristic curves of slope: $\mathrm{d}x/\mathrm{d}t = V \pm a$

$$\frac{\delta P}{\delta t} \pm \rho_f a \frac{\delta V}{\delta t} + \frac{2\rho_f a^2}{V \pm a}\left\{V\frac{\delta \varepsilon^r}{\delta t} \pm a \frac{\partial \varepsilon^r}{\partial t}\right\} \pm \rho_f a g \frac{\partial z}{\partial x} \mp \frac{4aT_f}{D} - 2\nu\rho_f a^2 \frac{\partial \dot{u}}{\partial x} = 0 \tag{5}$$

(b) Along the characteristic curve of slope $\mathrm{d}x/\mathrm{d}t = 0$

$$\frac{\partial \varepsilon^r}{\partial t} = \int_0^t \frac{\alpha D_m}{2e}\frac{\partial P(x, t-\tau)}{\partial t}\frac{\mathrm{d}J(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau \tag{6}$$

(c) Finite elements method for pipe wall

The two dynamic equations (3) and (4) of movement of pipe wall can be discretized into finite elements [16]. Using respectively the linear and cubic approximations for longitudinal and transverse displacement, the displacement vector in terms of nodal degree of freedom is: $q_e = [u_1, w_1, \theta_1, u_2, w_2, \theta_2]$, the shape functions

$$\left[N_e(x)\right] = \begin{pmatrix} L_1 & 0 & 0 & L_2 & 0 & 0 \\ 0 & N_1 & N_2 & 0 & N_3 & N_4 \\ 0 & N_1' & N_2' & 0 & N_3' & N_4' \end{pmatrix}$$

and the generic nodal relationship is $(u, w, \theta)_e = (N_e)q_e$.

So we have after discretization for the two dynamic vibration wall systems:

(c.1) Dynamic longitudinal vibrations

$$[m] = \int_0^L \rho_p A_p [N]^T [N]\,\mathrm{d}x$$

$$[c] = \int_0^L A_p E\tau\left[N'\right]^T\left[N'\right]\mathrm{d}x$$

$$[k] = \int_0^L EA_p\left[N'\right]^T\left[N'\right]\mathrm{d}x$$

$$\{f\} = -\int_0^L \left(A_f T_f + \frac{A_p D\nu}{2e}\frac{\partial P}{\partial x}\right)[N]^T\,\mathrm{d}x + \left([N]^T A_p E\left(\tau\dot{u}' + u'\right)\right)\Big|_0^L$$

(c.2) Dynamic transverse vibrations

$$[m] = \int_0^L m[N]^T[N]\,\mathrm{d}x$$

$$[c] = -\int_0^L 2m_f V\left[N'\right]^T\left[N'\right]\mathrm{d}x + \int_0^L EI\tau\left[N''\right]^T\left[N''\right]\mathrm{d}x$$

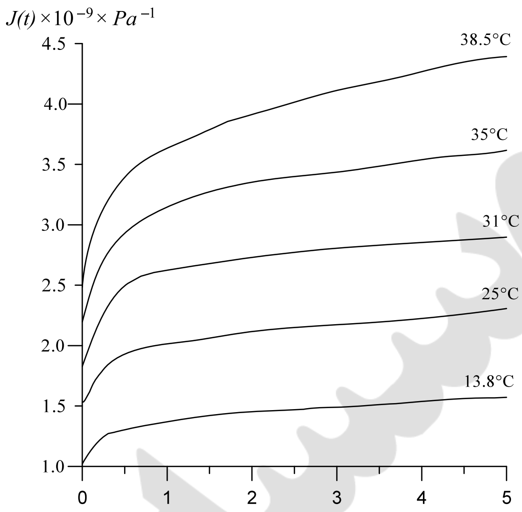

Fig. 2. Creep function versus temperature.

$$[k] = \int_0^L EI[N'']^T[N'']\,\mathrm{d}x - \int_0^L m_f V^2[N']^T[N']\,\mathrm{d}x$$

$$\{f\} = -\big([N]^T\big(EI\big(w''' + \tau\dot{w}'''\big) + m_f V^2 w' + 2m_f V\dot{w}\big)\big)\big|_0^L + \big([N']^T EI\big(w'' + \tau\dot{w}''\big)\big)\big|_0^L$$

The assembly of the two sub-systems gives the following global system in similar case:

$$[M]\ddot{X}(t) + [C]\dot{X}(t) + [K]X(t) = \{F(t)\} \tag{7}$$

where $X(t) = (u_1, w_1, \theta_1, u_2, w_2, \theta_2, \ldots, u_n, w_n, \theta_n)^T$ is the vector function of displacements of the pipe wall at nodes for the finite elements discretization.

To determinate the dynamic responses of pipe wall structure in a transient flow, the Newmark implicit temporal integration method is adapted because of its high accuracy and stability [16].

5. Application and results

The initial velocity was set equal to $V = 0.55$ m/s and the piezometric head at tank is $H = 0.55$ m, for transient flow of water at 25 °C; its cinematic viscosity 10^{-6} m^2/s and $\kappa = 2.1 \times 10^9$ Pa. The results presented below, represent the evolution of pressure versus time at the valve for instantaneous closing of valve (10 ms).

5.1. Classical model

For viscoelastic case, the results presented in Fig. 2 are obtained experimentally for polyethylene pipe line tested at the Laboratory of Fluid Mechanics of I.N.S.A Lyon [4].

This pipe is of data: $L = 43.1$ m, $D = 50$ mm, $e = 4.2$ mm, $\rho_m = 930$ kg/m^3, $\nu = 0.43$

$$J(t) = J_0 + \sum_{i=1}^{3} J_i\big(1 - \exp(-t/\tau_i)\big), \quad J_0 = 1.542 \times 10^{-9}\ \text{Pa}^{-1}, \quad J_1 = 0.754 \times 10^{-9}\ \text{Pa}^{-1}$$

$$J_2 = 1.046 \times 10^{-9}\ \text{Pa}^{-1}, \quad J_3 = 1.237 \times 10^{-9}\ \text{Pa}^{-1}, \quad \tau_1 = 0.89 \times 10^{-4}\ \text{s}, \quad \tau_2 = 0.022\ \text{s}, \quad \tau_3 = 1.864\ \text{s}$$

The viscoelastic behavior is even more pronounced as the temperature is high. At low temperatures, little change explains that one can admit behavior substantially elastic, but when temperature rises, the effect of viscoelasticity becomes dominant and must be taken into account in the calculations.

The results presented in Fig. 3 correspond respectively to:

- Graph a: measurement results [4].
- Graph b: theoretical calculation in elastic behavior under the conditions mentioned above, the losses being those corresponding to a steady friction term.
- Graph c: theoretical calculations in viscoelastic tube, the losses is also those of a steady friction term.

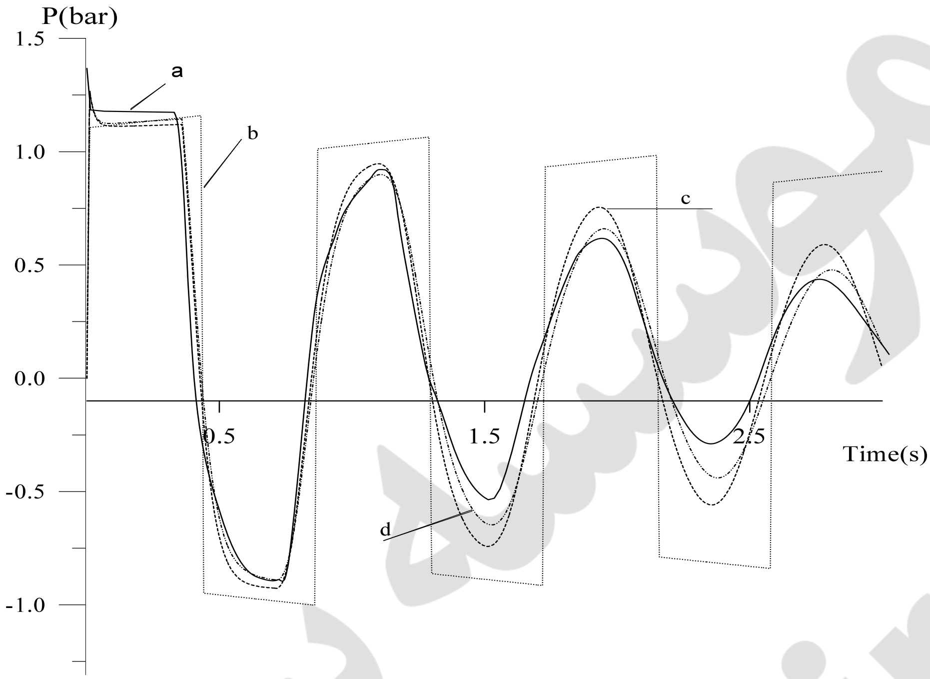

Fig. 3. Response of pressure at downstream end of polyethylene pipe on sudden closure of valve in classical model – comparison of theory and experiment at 25 °C.

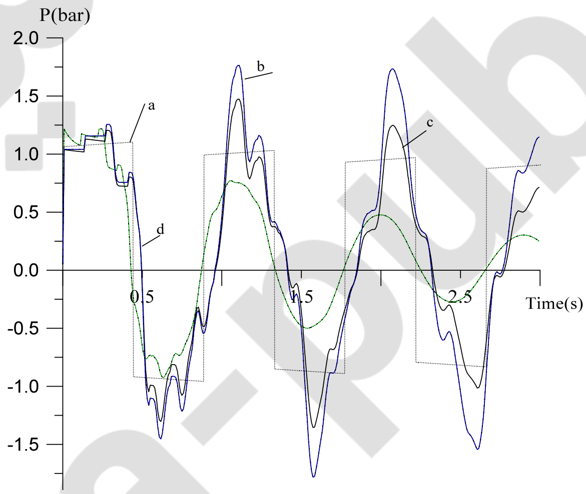

Fig. 4. Response of pressure at the downstream end of the polyethylene pipe on the sudden closure of a valve.

– Graph d: theoretical calculations in viscoelastic tube, still under the same conditions, but with taking into account the unsteady part of friction term [17,18]. This correction slightly improves the concordance between theory and experiment. It is found that viscoelasticity has a significant effect of depreciation that classical model in elastic case does not take into account.

5.2. *Fluid–structure interaction model*

In this case the calculations were conducted for a polyethylene pipe and a copper pipe. For a polyethylene pipe, as the first two relaxation times are small compared to the computation time, the conduit can be considered of Young modulus $E = 1/(J_0 + J_1 + J_2)$. Moreover, this part is limited at the level of losses, to the steady friction term.

The numerical results are shown in Figs. 4 and 5. In Fig. 4 corresponding to the polyethylene pipe, the graphs correspond respectively to:

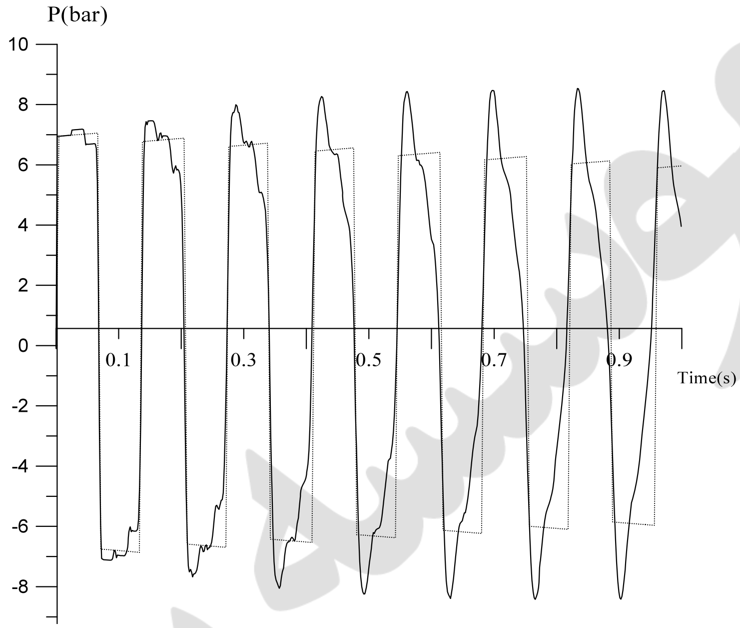

Fig. 5. Response of pressure at the downstream end of a rigid pipe (copper) on the sudden closure of a valve.

- Graph a: classical theory.
- Graphs b, c and d: model of fluid–structure interaction in elastic case: b and viscoelastic cases: c and d. The highlight the effects of fluid–structure interaction of fluid and pipe wall without or taking into account its viscoelastic character for arbitrary relaxation times $\tau = 1$ s and $\tau = 0.1$ s. We see, in elastic case, additional disturbances of pressure related to wave's propagation in the material of the pipe. These disturbances are added to the main disturbance in a complex interaction. The composite wave pressure could be of quite large magnitude. The viscosity of the fluid and the viscoelastic nature of the material have the effect of dissipation and damping of pressure waves.

Fig. 5 corresponds in the same conditions to the evolution of the pressure versus time at the valve, in a purely elastic behavior and relatively more rigid pipe (copper) with data $L = 26$ m, $D = 20$ mm, $e = 1$ mm, $E = 1.2 \times 10^{11}$ Pa, $\rho_m = 8920$ kg/m^3, $\nu = 0.33$.

We see a similar shape but with small pressure fluctuations. Our computer code is in good agreement with those recently found in the literature such as the work of Wiggert and Tijsseling [10].

6. Conclusion

We have attempted in this study, to give a numerical fluid–structure interaction code to calculate transients in elastic and viscoelastic pipes. This code includes, in addition to the rheology of the pipe wall, its dynamic behavior. Unlike the classical theory, this solver is able to accurately predict the phenomena of waterhammer. It highlights additional disturbances related to wave's propagation in the material of the pipe. These disturbances are added to the main disturbance in a complex interaction. The composite wave pressure could be of quite large magnitude. The viscosity of the fluid and the viscoelastic nature of the material have the effect of dissipation and damping of pressure waves. This code can be generalized for industrial pipes to predict the acoustic vibrations and in addition, be adapted to simulate, in hemodynamic, some arterial disease.

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